Cálculo Integral

Integrais Imediatos, Quase-Imediatos e por Decomposição

1. Calcule os integrais imediatos

a) $\displaystyle\int x^5\,dx$

b) $\displaystyle\int \frac{1}{x}\,dx$

c) $\displaystyle\int \sqrt[3]{x}\,dx$

d) $\displaystyle\int \cos(4x)\,dx$

e) $\displaystyle\int \operatorname{sen}\left(\frac{x}{3}\right)\,dx$

f) $\displaystyle\int \frac{1}{1+x^2}\,dx$

g) $\displaystyle\int e^{3x+2}\,dx$

h) $\displaystyle\int \sqrt{2+3x}\,dx$

i) $\displaystyle\int 5^{x^3}\,x^2\,dx$

j) $\displaystyle\int \frac{x}{1+x^2}\,dx$

k) $\displaystyle\int \frac{1}{1+4x^2}\,dx$

l) $\displaystyle\int \frac{x}{\sqrt{9-x^2}}\,dx$

m) $\displaystyle\int \sec^2(1-2x)\,dx$

n) $\displaystyle\int x^2\operatorname{cosec}(1-x^3)\operatorname{cotg}(1-x^3)\,dx$

2. Calcule os integrais

a) $\displaystyle\int \operatorname{tg}(3x)\,dx$

b) $\displaystyle\int \sec(5x)\,dx$

c) $\displaystyle\int \frac{1}{\sqrt{9-x^2}}\,dx$

d) $\displaystyle\int \frac{1}{4+9x^2}\,dx$

e) $\displaystyle\int \operatorname{sen}(2x)\sqrt{4-5\cos(2x)}\,dx$

3. Calcule os integrais

a) $\displaystyle\int \frac{7^{\operatorname{arcsen}(2x)}}{\sqrt{1-4x^2}}\,dx$

b) $\displaystyle\int \frac{\operatorname{sen}(2x)}{3+\cos^2(x)}\,dx$

c) $\displaystyle\int x\cos(x^2)\operatorname{sen}^2(x^2)\,dx$

d) $\displaystyle\int \frac{e^x}{\sqrt{4-e^{2x}}}\,dx$

4. Determine a primitiva $F(x)$ da função $f(x)=\sqrt{\dfrac{\sqrt{\operatorname{arcsen}(x)}}{1-x^2}}$, sabendo que $F(0)=1$.

5. Calcule

a) $\displaystyle\int \left(\sqrt{4+x^2}\right)'\,dx$

b) $\displaystyle\int \frac{d}{dx}\left(\operatorname{sen}(\sqrt[3]{x})\right)\,dx$

6. Calcule os integrais

a) $\displaystyle\int \frac{1+\sqrt[4]{x}}{x}\,dx$

b) $\displaystyle\int \frac{\cos(x)+\operatorname{cotg}(x)}{\sqrt[3]{\operatorname{sen}(x)}}\,dx$

c) $\displaystyle\int \frac{5x+2x^3}{1+x^4}\,dx$

d) $\displaystyle\int \frac{1+x-\sqrt{\operatorname{arcsen}(x)}}{\sqrt{1-x^2}}\,dx$

e) $\displaystyle\int \operatorname{sen}^2(5x)\,dx$

f) $\displaystyle\int \operatorname{sen}^3(3x-1)\,dx$

g) $\displaystyle\int \operatorname{tg}^2(-3x)\,dx$

h) $\displaystyle\int x\operatorname{cotg}^2(x^2)\,dx$

i) $\displaystyle\int \frac{1+\ln(x)}{x(1+\ln^2(x))}\,dx$

j) $\displaystyle\int \frac{x}{1+9x^4}\left(3x^2+10^{\operatorname{arctg}(3x^2)}\right)\,dx$

k) $\displaystyle\int \frac{\sec^2(2x)(1-3\operatorname{tg}(2x))}{\sqrt{1-\operatorname{tg}^2(2x)}}\,dx$

7. Determine a função integranda f, sabendo que

a) $\displaystyle\int f(x)\,dx = \operatorname{arctg}\left(\frac{2\operatorname{sen}(x)}{3}\right)+C$

b) $\displaystyle\int f(x)\,dx = \frac{x}{\sqrt{9-x^2}}+\arccos\left(\frac{x}{3}\right)+C$

Soluções

1. a) $\frac{1}{6}x^6+C$

b) $\ln|x|+C$

c) $\frac{3}{4}\sqrt[3]{x^4}+C$

d) $\frac{1}{4}\operatorname{sen}(4x)+C$

e) $-3\cos\left(\frac{x}{3}\right)+C$

f) $\operatorname{arctg}(x)+C$

g) $\frac{1}{3}e^{2+3x}+C$

h) $\frac{2}{9}\sqrt{(2+3x)^3}+C$

i) $\frac{1}{3\ln(5)}5^{x^3}+C$

j) $\frac{1}{2}\ln|1+x^2|+C$

k) $\frac{1}{2}\operatorname{arctg}(2x)+C$

l) $-\sqrt{9-x^2}+C$

m) $-\frac{1}{2}\operatorname{tg}(1-2x)+C$

n) $\frac{1}{3}\operatorname{cosec}(1-x^3)+C$

2. a) $-\frac{1}{3}\ln|\cos(3x)|+C$

b) $\frac{1}{5}\ln|\sec(5x)+\operatorname{tg}(5x)|+C$

c) $\operatorname{arcsen}\left(\frac{x}{3}\right)+C$

d) $\frac{1}{6}\operatorname{arctg}\left(\frac{3x}{2}\right)+C$

e) $\frac{1}{15}\sqrt{(4-5\cos(2x))^3}+C$

3. a) $\frac{1}{2\ln(7)}7^{\operatorname{arcsen}(2x)}+C$

b) $-\ln|3+\cos^2(x)|+C$

c) $\frac{1}{6}\operatorname{sen}^3(x^2)+C$

d) $\operatorname{arcsen}\left(\frac{e^x}{2}\right)+C$

4. $F(x) = \frac{4}{5}\sqrt[4]{\operatorname{arcsen}^5(x)} + 1$

5. a) $\sqrt{4+x^2} + C$

   b) $\operatorname{sen}(\sqrt[3]{x}) + C$

6. a) $\ln|x| + 4\sqrt[4]{x} + C$

   b) $\frac{3}{2}\sqrt[3]{\operatorname{sen}^2(x)} - \frac{3}{\sqrt[3]{\operatorname{sen}(x)}} + C$

   c) $\frac{5}{2}\operatorname{arctg}(x^2) + \frac{1}{2}\ln(1+x^4) + C$

   d) $\operatorname{arcsen}(x) - \sqrt{1-x^2} - \frac{2}{3}\sqrt{\operatorname{arcsen}^3(x)} + C$

   e) $\frac{1}{2}x - \frac{1}{20}\operatorname{sen}(10x) + C$

   f) $-\frac{1}{3}\cos(3x-1) + \frac{1}{9}\cos^3(3x-1) + C$

   g) $-\frac{1}{3}\operatorname{tg}(-3x) - x + C$

   h) $-\frac{1}{2}\operatorname{cotg}(x^2) - \frac{1}{2}x^2 + C$

   i) $\operatorname{arctg}(\ln(x)) + \frac{1}{2}\ln(1+\ln^2(x)) + C$

   j) $\frac{1}{12}\ln(1+9x^4) + \frac{1}{6\ln(10)}10^{\operatorname{arctg}(3x^2)} + C$

   k) $\frac{1}{2}\operatorname{arcsen}(\operatorname{tg}(2x)) + \frac{3}{2}\sqrt{1-\operatorname{tg}^2(2x)} + C$

7. a) $f(x) = \frac{6\cos(x)}{9+4\operatorname{sen}^2(x)}$

   b) $f(x) = \frac{x^2}{\sqrt{(9-x^2)^3}}$